Graph Theory Homework 5
Due: 26 June 2019 at 23:59pm as a PDF on Submitty
v1.1: Updated 21 June 2019

1. In class, we proved Euler's formula ($n - e + f = 2$) using edge contraction and induction on the number of vertices. Construct another proof of Euler's formula, instead this time doing induction on e and without using edge contraction.

2. Consider minimal non-planar graph G and graph $H = G - e$. Prove or disprove that we can guarantee $\exists e \in E(G)$ such that H is a maximal planar graph.

3. Prove graph G is outerplanar if and only if it doesn't contain a K_4 or $K_{2,3}$ subdivision.

4. Consider graph G. $\exists v \in V(G)$ such that $G - v$ has two components and $\forall v \in V(G) : d(v) = k$ for some arbitrary k. Prove that $\chi'(G) > k$. (**v1.1**: cut vertex, not cut edge)

5. Consider the $4 \times n$ chess board given below and a knight piece in the corner. As shown, a knight can only move in an "L" shape, across one box in one direction and two boxes perpendicular to that direction. We'll define a "knight tour" as the knight starting in the given bottom left corner, visiting all boxes on the board exactly once, and then returning to the original corner. Prove for what values of n that such a tour is possible.